Dear registrant,

Thank you for registering for *Mind-Benders for the Quarantined!* with Dr. Peter Winkler.

Here is a more serious push to help you solve the puzzle sent this past Sunday:

> **PUSH:** Notice that in a palindrome of even length, each digit appears an even number of times; in an odd-length palindrome, only the digit in the middle appears an odd number of times.

As a refresher, here is the puzzle along with the first hint:

> **PUZZLE: Concatenary Palindromes**
>
> Call a number "concatenary" if it is the concatenation of integers (written in base 10) from 1 to some larger integer. For example, 12 and 21 are concatenary; so are 132, 42153, and 129631185210741. To see that this last one qualifies, we space it out like this: 12 9 6 3 11 8 5 2 10 7 4 1.
>
> A number is a "palindrome" if it reads the same forwards and backwards, e.g., 7, 77, 747, 7447, or 2503361633052.
>
> What is the least number that is a concatenary palindrome?

SOLUTION: Suppose that our solution concatenates the numbers from 1 to *N*. Then each of the digits 1, 2, ..., 9, 0 must appear an even number of times in the numbers from 1 to *N*, with at most one exception, in order for those digits to be arrangeable into a palindrome.

So a natural first task is to determine the least *N* with this property, and it turns out to be 19. The numbers from 1 to 19 contain 12 1's, 2 of each digit from 2 to 9, and just one 0 (which would thus have to be in the middle of our concatenary palindrome).

We don't know yet whether there is a concatenary palindrome made from the numbers 1 through 19, but let's be optimistic and try to construct the smallest one we can. We'd want to start the number with the digit 1, and thus it must also end with 1. Can we begin it with two 1's? Yes, but only if we end it with the number eleven, and begin it with the number 1 followed by something in the teens. So our number looks like "1 1*x* ... 0 ... *x* 11" for some digit *x*. So far so good.

We can't have $x = 1$, but $x = 2$ is OK, so our number becomes "1 12 *y* ... 0 ... *y* 2 11" for some digit *y*. Can *y* be 1? No, because the *y* on the right side can only belong to the number one or the number eleven and we've already used those. But it can be 3, provided that on the right it's part of the number 13. So now we have "1 12 3 1*z* ... 0 ... *z* 13 2 11," where *z* is some digit — which can be taken to be 4.

Proceeding in this manner we end up with

1 12 3 14 5 16 7 18 9 10 19 8 17 6 15 4 13 2 11,

[Puzzle proposed by subscriber (and puzzle book author!) Rodolfo Kurchan, who found it on the web and traced it through Carlos Rivera to Dmitry Kamenetsky, who published it in a Russian forum.]