FOREST OF NUMBERS: (Bosque de Números)

HISTORY OF THE PROBLEM:

The Forest of Numbers started with a puzzle of **Diego Kovacs** in the magazine **Humor 266**, **La Odisea del Ingenio** puzzle column**, May 1990)**

In the **Magazine Aprox**, in his only 2 numbers from **October 1990 and January 1991**, released some results.

In **Magazine El Acertijo** Number 5, **April 1993** appeared some problems

https://el-acertijo.blogspot.com/2008/06/el-acertijo-05-pagina-08.html

https://el-acertijo.blogspot.com/2008/06/el-acertijo-05-pagina-09.html

https://el-acertijo.blogspot.com/2008/06/el-acertijo-05-pagina-18.html

and some improvements on **magazine El Acertijo** Number 7, **August 1993**

https://el-acertijo.blogspot.com/2008/07/el-acertijo-07-pagina-15.html

PROBLEM 1: by Jaime Poniachik and Ivan Skvarca

Start with an infinite square grid. Each cell has eight neighbors. Place the numbers 1, 2, ..., n anywhere. Now place the numbers n+1, n+2, ..., m in order, subject to the rule that when you place k, the sum of its neighbors must equal k.

OEIS: https://oeis.org/A350627

a) 1-2 = 10

	1	4	
9	3		6
	5	2	8
	7		10

b) 1-2-3 = 22 by Daniel Valdano

			21		18		
			14	7	11		
			6	1		3	19
	20	8		5	4		16
	10	2	15		9	13	
22	12		17				

c) 1-2-3-4 = 30 by Daniel Valdano

			15		26					
28	17		7	8	18			24	27	
	11	6	1		10		21		3	
20		5		23	12	2		19	22	25
29	9	4				14	16			
		13				30				

d) 1-2-3-4-5 = 36 by Giorgio Vecchi

								14		34
35	31				36		24		12	2	32
	4	27		21		15	9		10		30
		23		16	5		1	8	18	28	
	26		19		11	6	7				
	29	3	22		17		13	20			
			25				33				

e) 1-2-3-4-5-6 = 44 by Dmitry Kamenetsky

		32		31						
		29	3	28						
		26		25						
35	23				22	42				
	12	11	24	2	20					
	39	1	10		18			44		
	8	7		16		37		40	4	34
	15		6			19	14	9	13	17
	36	21	27	33		38	5	41		30
						43				

Square Fields:

PROBLEM 2: by Rodolfo Kurchan

As problem 1 but using numbers from 1 to N in a NxN square

https://oeis.org/A352814

a) 2x2 = 3

1	3
	2

b) 3x3 = 8

8	4	7
1	3	
6	2	5

c) 4x4 = 12 by Hector San Segundo

	10	3	8
	6	1	4
	2		5
11	9	7	12

d) 5x5 = 19 by Pontus von Brömssen

5	6	7	8	18
11		1		10
14		19	2	16
	3	9	4	
15	12		13	17

e) 6x6 = 25 by Giorgio Vecchi

22	1	15	19		20
7	14			4	16
	6		21		12
17		9		8	25
	11		3	5	
24	13	2	10	18	23

e) 7x7 = 34 by Giorgio Vecchi

33	30			34		31
	3	27	14	13	7	24
18	9		1		5	12
6			11	32		17
16	8	10		15		21
26	2	20		19	4	25
28		22		23		29

f) 8x8 = 37 by Dmitry Kamenetsky

34		36		27	16	5	31
22	12	24			11		26
	10	2			6	21	
28		8	19			4	25
	18	9		32			29
33		1	13		35		
	15	14		3			7
			17	20	23	30	37

g) 9x9 = 45 by Dmitry Kamenetsky

				28	37	41		29
44	39	34	19	9			4	25
	5		10		40	17	21	
36	31	26	11	1	13			
				12		27	30	33
			20		14			3
	23	8	43		16	2		35
45		15			18		32	
	22	7			42	24	6	38

		29	38				31	39	
	20	9			47	23	8		
33	11		37		24		15	30	
46	2		27	1	25		7		42
48				26	52		41	12	
							5	17	51
50		10		45		49	44	22	
36	14		13	32					28
4	18		3	16				34	6
	43	21		19	35			40	

PROBLEM 3: by Rodolfo Kurchan

As problem 2 but for **Transparent Rooks** (a new number is the sum of all numbers in horizontal and vertical). You always start with numbers 1 and 2

a) 2x2 = 3 by Rodolfo Kurchan

1	3
	2

b) 3x3 = 6 by Rodolfo Kurchan

1	4	3
		5
	6	2

c) 4x4 = 8 by Rodolfo Kurchan

1	4	8	3
7			
6			5
			2

Pontus von Brömssen wrote: **I can prove that 8 is optimal for all N>=4.**

PROBLEM 4: by Rodolfo Kurchan

As problem 3 but for **Opaque Rooks** (a number stops the action of the numbers behind). You always start with numbers 1 and 2

https://oeis.org/A353103

a) 2x2 = 3 by Rodolfo Kurchan

1	3
	2

b) 3x3 = 7 by Rodolfo Kurchan

1	4	3
		5
7	6	2

c) 4x4 =9 by Rodolfo Kurchan

1	5	4	3
9			
8			
7		6	2

d) 5x5 =12 by Rodolfo Kurchan

1			**4**	**3**
6				**5**
			11	**7**
				9
8	**10**	**12**		**2**

e) 6x6 =16 by Rodolfo Kurchan

1	**7**	**6**	**5**	**4**	**3**
13					
12					
11			**16**		
10					**15**
9		**8**		**14**	**2**

f) 7x7 = 21 by Giorgio Vecchi

1	5	9	4	7	18	3
13				20		
12		21				
	19		14			
11			10			15
			16			
17			6		8	2

g) 8x8 = 25 by Giorgio Vecchi

1	22	4	15	11	18	7	3
9	17						8
				24			13
						12	5
		14					21
19		10	25				
		16					
	23	6				20	2

h) 9x9 = 32 by Giorgio Vecchi

1		**26**	**4**	**11**	**18**	**7**	**32**	**3**
	31		**14**					**25**
29						**19**		
9	**17**							**8**
				24				**13**
	27		**10**					**28**
					30	**12**		**5**
			16					**23**
15		**21**	**6**			**20**	**22**	**2**

i) 10x10 = 37 by Giorgio Vecchi

1		30	4	15	11		7	32	3
	35		18						29
24			14				33		
9	17	25							8
				36					21
			37						13
19			10						28
31							12		5
			16		27				34
	23		6			26	20	22	2

PROBLEM 5: by Rodolfo Kurchan

As problem 3 but for **Transparent Queens** (a new number is the sum of all numbers in horizontal, vertical and diagonally). You always start with numbers 1 and 2

https://oeis.org/A353070

a) 2x2 = 3 by Rodolfo Kurchan

1	3
	2

b) 3x3 = 5

1	2	
3		5
4		

c) 4x4 = 8 by Gustavo Piñeiro

3		5	2
4			
			8
1	7		6

d) 5x5 =10 by Pontus von Brömssen

1		2	8	
9	3	5		
				6
	10			
	7			4

e) 6x6 =13 by Pontus von Brömssen

1	9		8		
			7		12
10					
		6			2
5				11	
4		13			3

f) 7x7 =15 by Pontus von Brömssen

	8	2	6			
	11	2				
				5		15
3			1			10
14						
7		12				
4				9	13	

g) 8x8 =17 by Pontus von Brömssen

1	3		12	6		2	
4						11	
		7		15			
							14
	13						
10						5	
		16			9		
					17		8

h) 9x9 =19 by Giorgio Vecchi

1	**2**			**3**	**12**			
								14
5				**15**				
10								
4					**6**		**19**	
			16					
	13	**11**						
						17	**9**	**8**
		18		**7**				

PROBLEM 6: by Rodolfo Kurchan

As problem 5 but for **Opaque Queens** (a number stops the action of the numbers behind). You always start with numbers 1 and 2

https://oeis.org/A353093

a) 2x2 = 3

1	3
	2

b) 3x3 = 6

2	6	3
	1	
	5	4

c) 4x4 = 10 by Giorgio Vecchi

8	4		9
1	3		5
6		2	
		10	7

d) 5x5 = 13 by Daniel Valdano

12	4	8		
7	1	3		10
		6		
	9	2	13	5
	11			

e) 6x6 = 16 by Daniel Valdano

16	11	6			14
1	4		15	3	5
	7			2	10
					12
		8			
13				9	

f) 7x7 = 20 by Giorgio Vecchi

15	4	20		12		
10	1	3				14
17						
	6	5	16	2		9
	11					
				7	8	19
	13	18				

g) 8x8 = 23 by Giorgio Vecchi

13	8	15	18				
1	4	3			10		
							21
19	6	12					
9		2		22	5	20	
	17						
							16
23		14			7		11

PROBLEM 7: by Giorgio Vecchi

As problem 3 but for **Transparent Bishops** (a new number is the sum of all numbers in diagonally). You always start with numbers 1 and 2

a) 3x3 = 5 by Giorgio Vecchi

1		2
	3	
5		4

b) 4x4 = 6 by Giorgio Vecchi

1		2	
	3		6
5		4	

c) 5x5 = 8 by Giorgio Vecchi

4		6		2
	1		5	
7		3		
			8	

It seems that 8 is optimal for all N>=5.

PROBLEM 8: by Giorgio Vecchi

As problem 5 but for **Opaque Bishops** (a number stops the action of the numbers behind). You always start with numbers 1 and 2

a) 3x3 = 3 by Giorgio Vecchi

1		**2**
	3	

b) 4x4 = 4 by Giorgio Vecchi

1			
	4		
		3	
			2

c) 5x5 = 7 by Giorgio Vecchi

		3		
	4		5	
1				2
			6	
		7		

d) 6x6 = 8 by Giorgio Vecchi

		3			
	4				
1				5	
	8				2
				6	
			7		

e) 7x7 = 9 by Giorgio Vecchi

	8					
1						
	6		7			
		5				
			9			
				4		2
					3	

f) 8x8 = 11 by Giorgio Vecchi

			3				
		4		5			
					7		
1				11		9	
	6						2
					10		
				8			

g) 9x9 = 12 by Giorgio Vecchi

		6						
1		12		11		9		
	4							
								2
			7				5	
				10		8		
					3			

h) 10x10 = 14 by Giorgio Vecchi

i) 11x11 = 16 by Giorgio Vecchi

					3					
			4							
		5								
	6								14	
1										2
	12								15	
		11		16				13		
			10				7			
				9						
					8					

				9							
			10								
		11									
	12		**18**			8					
1											
	7					15		14			
		13							16		
			6								**2**
			5								
				4							
							17				
						3					

k) 13x13 = 21 by Giorgio Vecchi

			11					20				
		12										
	13								6			
1		18						16		8		
	5				21		10				17	
		14										2
			9									
				4						15		
					19							
						7						
							3					

l) 14x14 = 23 by Giorgio Vecchi

					3								
				7									
			11				21						
		4						8					
	22		14						13				
1				10						18			
											5		
		20		17						12		23	
			9										2
									16		19		
										6			
								15					

m) 15x15 = 25 by Giorgio Vecchi

									6					
				19				16						
		9				10								
	24		17										20	
1								21						2
	22		14								12		23	
		4										5		
			15											
				11						13				
					18				8					
						7		25						
							3							

n) 16x16 = 28 by Giorgio Vecchi

					19				23						
										6					
									16						
		9						10							
	24		26												
1				17				27		21				20	
															2
		22		14											
			4								12		28		
				15								5			
					11						13				
						18				8					
							7		25						
								3							

Minimum and Maximum Forest in Square fields

Same rules as in Square Fields, but now numbers do not necessarily have to be consecutive.

The goal is to find the minimum and maximum number for each NxN square.

Repeated numbers are not allowed

We start with numbers 1 and 2

(The original idea was starting with numbers from 1 to N in a NxN square by **Diego Bracamonte** for the maximum possible and **Rodolfo Kurchan** for the minimum possible)

PROBLEM 9:

https://oeis.org/A352621

Minimum

a) 2x2 = 6 **by Rodolfo Kurchan**

1	3
6	2

b) 3x3 = 12 **by Rodolfo Kurchan**

12	7	10
4	1	2
8	3	6

c) 4x4 = 36 **by Rodolfo Kurchan**

33	21	5	28
11	1	4	19
24	10	3	7
36	2	22	32

d) 5x5 = 68 **by Dmitry Kamenetsky**

32	18	63	30	56
13	1	4	10	16
9	3	39	6	54
36	5	2	8	14
12	7	22	46	68

e) 6x6 = 140 **by Dmitry Kamenetsky**

124	72	14	38	120	37
28	24	6	8	10	27
75	4	53	2	131	17
18	1	3	5	7	55
89	13	9	57	12	19
35	22	122	21	140	31

f) 7x7 = 292 **by Dmitry Kamenetsky**

93	241	40	117	50	252	95
68	25	15	256	12	38	57
43	247	10	5	7	19	267
65	18	3	2	125	56	97
150	4	1	6	8	22	285
47	16	11	289	14	44	66
90	27	54	137	58	292	110

g) 8x8 = 502 **by Dmitry Kamenetsky**

356	244	112	34	48	85	502	101
68	44	20	14	420	215	37	64
267	24	230	6	8	10	27	225
42	89	4	29	2	76	17	80
344	18	1	3	5	7	383	36
53	142	13	9	300	12	19	136
379	35	22	108	21	135	31	50
92	57	265	43	359	52	349	81

h) 9x9 = 787 **by Dmitry Kamenetsky**

183	115	378	122	49	145	73	337	132
761	68	47	26	556	23	743	50	82
104	291	21	121	5	65	18	32	338
644	36	15	6	1	4	14	71	103
162	51	203	9	70	3	7	501	235
757	75	119	11	2	34	10	17	44
136	333	24	13	334	12	281	27	247
628	61	37	161	25	76	39	66	93
787	98	419	62	464	140	678	357	159

i) 10x10 = 1391 **by Dmitry Kamenetsky**

424	212	492	147	95	238	100	398	149	241
131	81	1200	52	839	43	1001	57	92	627
348	50	31	135	21	108	22	35	1349	145
86	1135	19	12	300	9	13	142	53	198
318	36	247	7	5	3	1	18	1159	455
116	80	17	76	2	29	4	89	42	162
425	931	27	10	8	6	186	24	515	120
165	64	37	195	38	14	20	1131	78	1231
985	101	1176	75	1391	106	34	54	220	298
378	277	176	251	572	140	422	88	1178	518

PROBLEM 10:

Maximum

a) 2x2 = 6 **by Rodolfo Kurchan**

1	3
6	2

b) 3x3 = 90 **by Claudio Meller**

1	47	90
3	11	32
2	5	16

c) 4x4 = 3676 **by Rodolfo Kurchan**

1	2	1842	3676
3	6	620	1214
9	18	150	444
27	54	72	222

d) 5x5 = 527.024 **by Dmitry Kamenetsky**

2	11	17	51	153
3	6	34	102	306
1	45440	24282	442	850
146614	101173	21114	2584	1292
247787	527024	10336	7752	3876

e) 6x6 = `229.726.440` **by Dmitry Kamenetsky**

459	918	2550	3876	11628	34884
153	306	1326	7752	23256	69768
51	102	229726440	115219591	100776	193800
17	34	75512472	38984857	589152	294576
11	6	21798671	14728802	1767456	883728
2	3	1	7069825	5302368	2651184

PROBLEM 11:

Forest of transparent Rooks in Square fields

You start with numbers 1 and 2

A new number is the sum of all numbers in horizontal and vertical.

Minimum

a) 2x2 = no solution because repeat number 3

b) 3x3 = 30 **by Rodolfo Kurchan**

1	4	3
15	**30**	5
9	6	**2**

c) 4x4 = 119 **by Rodolfo Kurchan**

11	7	28	**119**
4	**1**	3	8
53	79	**2**	10
26	6	5	55

PROBLEM 12:

Forest of transparent Rooks in Square fields

You start with numbers 1 and 2

A new number is the sum of all numbers in horizontal and vertical.

Maximum

a) 2x2 = no solution because repeat number 3

b) 3x3 = 37 **by Rodolfo Kurchan**

1	15	3
6	11	5
37	28	2

c) 4x4 = 1058 **by Rodolfo Kurchan**

1	70	45	3
11	21	42	10
1058	96	188	5
757	466	277	2

PROBLEM 13:

Forest of opaque Rooks in Square fields

You start with numbers 1 and 2

A new number is the sum of numbers in horizontal and vertical that a rook can see (maximum 4 different numbers, minimum 2)

A number stops the action of the numbers behind.

Minimum

a) 2x2 = no solution because repeat number 3

b) 3x3 = 21 **by Rodolfo Kurchan**

1	4	3
10	9	5
21	11	2

c) 4x4 = 78 **by Rodolfo Kurchan**

1	5	4	3
9	14	24	40
8	35	78	13
7	48	6	2

PROBLEM 14:

Forest of opaque Rooks in Square fields

You start with numbers 1 and 2

A new number is the sum of numbers in horizontal and vertical that a rook can see (maximum 4 different numbers, minimum 2)

A number stops the action of the numbers behind.

Maximum

a) 2x2 = no solution because repeat number 3

b) 3x3 = 35 **by Rodolfo Kurchan**

1	14	3
6	35	5
8	10	2

c) 4x4 = 332 **by Rodolfo Kurchan**

1	4	7	3
11	82	67	40
21	230	332	26
10	101	9	2

PROBLEM 15:

Forest of transparent Queens in Square fields

You start with numbers 1 and 2

In **Transparent Queens** a new number is the sum of all numbers in horizontal, vertical and diagonally.

Minimum

a) 2x2 = 6 **by Rodolfo Kurchan**

1	3
6	2

b) 3x3 = 41 **by Rodolfo Kurchan**

41	6	20
3	1	2
4	18	7

PROBLEM 16:

Forest of transparent Queens in Square fields

You start with numbers 1 and 2

In **Transparent Queens** a new number is the sum of all numbers in horizontal, vertical and diagonally.

Maximum

a) 2x2 = 6 **by Rodolfo Kurchan**

1	3
6	2

b) 3x3 = 117 **by Rodolfo Kurchan**

1	27	3
15	117	55
6	8	2

PROBLEM 17:

Forest of opaque Queens in Square fields

You start with numbers 1 and 2

In **Opaque Queens** a new number is the sum of numbers in horizontal, vertical and diagonally that the Queen can see. (maximum 8 different numbers, minimum 2)

A number stops the action of the numbers behind.

Minimum

a) 2x2 = 6 **by Rodolfo Kurchan**

1	3
6	2

b) 3x3 = 12 **by Rodolfo Kurchan**

8	3	6
4	1	2
12	7	10

c) 4x4 = 36 **by Rodolfo Kurchan**

33	21	5	28
11	1	4	19
24	10	3	7
36	2	22	32

PROBLEM 18:

Forest of opaque Queens in Square fields

As problem 15 but you try to reach maximum number possible

Maximum

a) 2x2 = 6 **by Rodolfo Kurchan**

1	3
6	2

b) 3x3 = 117 **by Rodolfo Kurchan**

1	27	3
15	117	55
6	8	2

PROBLEM 19: by Diego Kovacs

Back to the origin (this is first puzzle idea)

As problem 1, but you have a river that divide the 2 forest.

Place some numbers from 1 to N between the 2 different region to reach the biggest possible number without repeating any number in any region.

All numbers should appear in order and be consecutive

a) 1-5 = 27 by Rodolfo Kurchan

23				24					
21	2	16	5	6	18		15		
19		8	1	12			11	4	14
	17	9		13	25			7	3
	26		22				27	20	10

STONES ON SQUARE FIELDS

NEW VARIATIONS by Rodolfo Kurchan

PROBLEM 1:

Same rules as Stones on an infinite chessboard but you have to find solutions to complete squares fields.

Original Video Stones on an Infinite Chessboard – Numberphile **by Neil Sloane**
https://www.youtube.com/watch?v=m4Uth-EaTZ8

Put the lowest possible stones to have the most quantity of consecutive numbers starting from 2 in a NxN square.

Repeated numbers are not allowed

There are no solutions for boards 2x2 and 3x3

4x4 = 10 **by Pontus von Brömssen**

1	9	1	8
5	1	1	6
2	1	1	3
7	4	10	1

5x5 = 16 **by Pontus von Brömssen**

2	1	1	16	7
5	1	1	3	4
8	1	15	1	9
11	1	6	1	12
13	1	10	1	14

MINIMUM AND MAXIMUM:

Put n stones in a NxN square to reach the minimum and maximum possible numbers.

Repeated numbers are not allowed

Numbers may not be consecutives

PROBLEM 2:

Minimum

a) 2x2 = 4 **by Rodolfo Kurchan**

1	**2**
4	**1**

b) 3x3 = 10 **by Rodolfo Kurchan**

10	**2**	**4**
7	**1**	**1**
1	**3**	**5**

c) 4x4 = 17 **by Rodolfo Kurchan**

16	11	5	13
1	4	1	7
6	1	1	14
17	10	2	3

d) 5x5 = 29 **by Claudio Meller**

12	1	24	9	17
1	10	1	3	5
27	7	29	2	25
8	1	4	1	14
28	19	6	11	26

e) 6x6 = 65 **by Giorgio Vecchi**

1	1	17	58	20	62
60	25	1	15	5	37
32	1	4	2	3	9
59	6	31	63	1	57
13	7	1	10	11	33
49	29	8	51	21	65

PROBLEM 3:

Maximum

a) 2x2 = 4 **by Rodolfo Kurchan**

1	2
4	1

b) 3x3 = 49 **by Claudio Meller**

1	14	49
4	9	26
1	2	1

c) 4x4 = 1362 **by Claudio Meller**

1362	448	162	81
683	231	55	26
1	3	10	16
1	1	5	1

d) 5x5 = 149.720 **by Giorgio Vecchi**

1	**1**	**1**	**74861**	**149720**
7	**5**	**1**	**24954**	**49905**
26	**14**	**1**	**8538**	**16413**
40	**81**	**661**	**1969**	**5906**
121	**242**	**323**	**984**	**2953**

PROBLEM 4:

Prime Numbers

Original Video Stones on an Infinite Chessboard – Numberphile **by Neil Sloane**
https://www.youtube.com/watch?v=m4Uth-EaTZ8

As problem infinite stone of prime numbers but you start with only one number 1 and then each stone has a value of 2.

You have to try to put most quantity of consecutive primes

https://www.primepuzzles.net/puzzles/puzz_1079.htm

https://www.primepuzzles.net/puzzles/puzz_1085.htm

a) with 1 stone = 13 by Rodolfo Kurchan

		1
	11	3
13	2	5
		7

b) with 2 stones = 23 by Rodolfo Kurchan

		1	23	
	11	3		19
13	2	5	17	2

		7		

c) with 3 stones = 31 by Rodolfo Kurchan

		1	23	
	11	3		19
13	2	5	17	2
	29	7	31	
2				

d) with 4 stones = 41 by Dmitry Kamenetsky

	2	37	31		1	
23	2		2	29		3
	19	17	13	11	2	5
			41		7	

e) with 5 stones = 53 by Giorgio Vecchi

	47		31	
43	2	2	29	
41		23	3	1
2	37	5	11	
	7		2	13
			17	53
			19	2

f) with 6 stones = 61 by Giorgio Vecchi

			31	2	2	61	
				29		59	
	47	37	23		2	53	2
3	2	5	7	11	13	17	19
	1		2		41		
				43			

g) with 7 stones = 73 by Giorgio Vecchi

					61		47			
	7		71		59		2	2	43	2
2	5	41	13	17	19	23	29	2	37	
	3	11	2	53	2	73		31		
		1	67							

h) with 8 stones = 89 by Giorgio Vecchi

				19	2	73		
				17			71	
		29	13	2	23	2	67	2
7	5	11	59		31	2	61	
	2	3	1		37	2	53	
		89	83	79	41	2	47	
						43		

i) with 9 stones = 97 by Dmitry Kamenetsky

		19	89					
61	2	53	17					
	59	2	13	2				
		2	7	23		97	2	43
		67	47	1	29	31	37	41
			11	3	71	2	2	
			2	5		73	79	83
							2	

i) with 10 stones = 103 by Giorgio Vecchi

				103	2	
				101		47
	2		97		2	43
	23	29	31	37	41	
19	2		2		2	
17	53	59	61	67	2	
13	2				71	
11		5	89	2	73	
2	7		3	79		
			83	1		

From primepuzzles.net:

Fred Schneider wrote:

I took a different approach. I just tried to see how far I could go. The best I could come up with was 54 stones (from prime 3 to prime 211). It's the best solution I found. I think it could be optimal but I'm not positive.

Up to 211 — 54 Green stones

										2	2		2							
								2	2	163	167	173	2	2						
									157				179							
7	5	3	1				2	2	151				181	2	2					
	2	11						2	149				191	2						
		13					2	2	139				193	2	2					
2	17							2	137				197							
2		19	23	2	2				131		2				199	2	2			
			2	29		37	41	43	47	2					211	2				
				31	2	2			53	2					2	2	2	2	2	
									2	59								97		
										2	61			2	2	83	89	2	101	
													67	2	79	2		2		103
													2	71					107	2
													73						109	
															2	2	113			
															2	127	2			
															2	2	2			