## COLORED POLYOMINOES: IN PREPARATION

Last update September 29, 2023

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COLORED POLYOMINOES WITHOUT GAPS, images by George Sicherman


## 1\}

## https://oeis.org/draft/A365835

For every cell of a polyomino let $b$ be the number of cells that are in the same row or in the same column (including itself). Cells beyond gaps do not count. a(n) is the sum of the b values of all cells of all free polyominoes with n cells.

## 1, 4, 16, 62, 204, 776, 2936, 12030, 48783, 202734, 839239, 3489810, 14462593

a(6)-a(9) from George Sicherman, Sep 202023
a(6)-a(9) corrected and a(10)-a(13) added by Pontus von Brömssen, Sep 212023

## 2\}

## https://oeis.org/draft/A365860

For every cell of a polyomino let c be the number of cells that are in the same row or in the same column (including itself). $\mathrm{a}(\mathrm{n})$ is the sum of the c values of all cells of all free polyominoes with $n$ cells.

1, 4, 16, 62, 206, 790, 3042, 12648, 52181, 220372, 927333, 3917738, 16491489
a(7)-a(9) from George Sicherman, Sep 202023
a(10)-a(13) from Pontus von Brömssen, Sep 212023

## 3\}

## https://oeis.org/draft/A365906

Irregular triangle $T(n, k)$ read by rows, $n>=1, k>=1$, in which row $n$ lists in nonincreasing order the sum of the $b$ values (described in A365835) of the cells of every free polyomino with $n$ cells.

```
1, 4, 9, 7, 16, 12, 12, 12, 10, 25, 19, 19, 17, 17, 17, 17, 15, 15,
15, 15, 13, 36, 28, 28, 28, 24, 24, 24, 24, 24, 24, 24, 22, 22, 22,
22, 22, 22, 22, 22, 22, 22, 22, 20, 20, 20, 20, 20, 20, 20, 18, 18,
18, 18, 18, 16, 49, 39, 39, 39
1,
4,
9, 7,
16, 12, 12, 12, 10,
25, 19, 19, 17, 17, 17, 17, 15, 15, 15, 15, 13,
36, 28, 28, 28, 24, 24, 24, 24, 24, 24, 24, 22, 22, 22, 22, 22, 22,
22, 22, 22, 22, 22, 20, 20, 20, 20, 20, 20, 20, 18, 18, 18, 18, 18,
16,
49, 39, 39, 39 ....
```


## 3a\} number of different terms that appear in each row $n$ of A365906.

Irregular triangle $T(n, k)$ read by rows, $n>=1, k>=1$, in which row $n$ lists in nonincreasing order the sum of the $b$ values (described in A365835) of the cells of every free polyomino with $n$ cells.
$1,1,2,3,5,7,10,14,19,25,32,40,49, \ldots$.
To confirm
formula next term $a(n\}=a(n-1\}+(n-3\}$ for $a(n)>3$

## 4\}

Quantity of polyominoes of each value (up to pentominoes\}

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 0 | 3 | 1 | 0 | 4 | 1 | 4 | 0 | 2 |  |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |

## 5\}

In what term DOES THE FIRST NUMBER OF EACH VALUE APPEAR?
Will all the numbers appear?

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2 | 1 | 16 | 12 | 15 |  |  |  |  |  |  |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |  |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |  |


| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

6\}
SUM OF THE QUANTITY OF DIFFERENT COLORS THAT EACH PIECE HAS PER POLYOMINO

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 1 | 3 | 10 | 29 | 102 |  |  |  |  |

## 7\}

NUMBER OF DIFFERENT COLORS THAT EACH PIECE HAS PER POLIOMINO IN TRIANGULAR SHAPE SHOWING ALL THE PIECES

1,
1,
1, 2,
1, 1, 2, 3, 3
$1, \quad 2, \quad 2, \quad 2, \quad 2, \quad 2, \quad 3, \quad 3, \quad 3, \quad 3, \quad 3, \quad 3$

## 8\}

MAXIMUM NUMBER OF COLORS THAT EACH POLYOMINO HAS

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 | 6 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 7 | 8 | 8 | 9 | 9 | 10 |  |  |  |  |

solutions from $\mathrm{a}(8)$ to $\mathrm{a}(16)$ by George Sicherman
Example of heptomino with 5 different number

|  | 4 |  |
| :--- | :--- | :--- |
| 3 | 6 | 3 |
|  | 4 |  |
|  | 5 | 2 |

Example of 12-omino with 8 different colors/numbers by George Sicherman


| 4 |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 4 |  |  |  |  |  |
| 5 | 4 |  |  |  |  |
| 9 | 8 | 6 | 7 | 6 | 6 |
|  | 3 |  | 2 |  |  |

Example of 14-omino with 9 different colors/numbers

|  | 4 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 4 |  |  |  |  |  |
| 4 | 6 | 5 |  |  |  |  |
| 8 | 10 | 9 | 7 | 8 | 7 | 7 |
|  |  | 3 |  | 2 |  |  |



Example of 12-omino with 8 different colors/numbers by George Sicherman


Is it true that cannot be an increase of more than 1 from 1 n omino to $n+1$-omino?

9\}

NUMBER OF DIFFERENT POLYOMINOES WITH MAXIMUM NUMBER OF COLORS FOR EACH POLYOMINO

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 1 | 1 | 2 | 6 | 7 | 12 | 99 | 83 | 692 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 332 | 56 | 3.356 | 840 | 10.060 | 1983 |  |  |  |  |

solutions from a(7) to a(16) by George Sicherman

| $n$ | name | free |  |  | one-sided | fixed |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | total | with holes | without holes |  |  |
| 1 | monomino | 1 | 0 | 1 | 1 | 1 |
| 2 | domino | 1 | 0 | 1 | 1 | 2 |
| 3 | tromino | 2 | 0 | 2 | 2 | 6 |
| 4 | tetromino | 5 | 0 | 5 | 7 | 19 |
| 5 | pentomino | 12 | 0 | 12 | 18 | 63 |
| 6 | hexomino | 35 | 0 | 35 | 60 | 216 |
| 7 | heptomino | 108 | 1 | 107 | 196 | 760 |
| 8 | octomino | 369 | 6 | 363 | 704 | 2,725 |
| 9 | nonomino | 1,285 | 37 | 1,248 | 2,500 | 9,910 |
| 10 | decomino | 4,655 | 195 | 4,460 | 9,189 | 36,446 |
| 11 | undecomino | 17,073 | 979 | 16,094 | 33,896 | 135,268 |
| 12 | dodecomino | 63,600 | 4,663 | 58,937 | 126,759 | 505,861 |
| OEIS sequence |  | A000105 | A001419 | A000104 | A000988 | A001168 |

