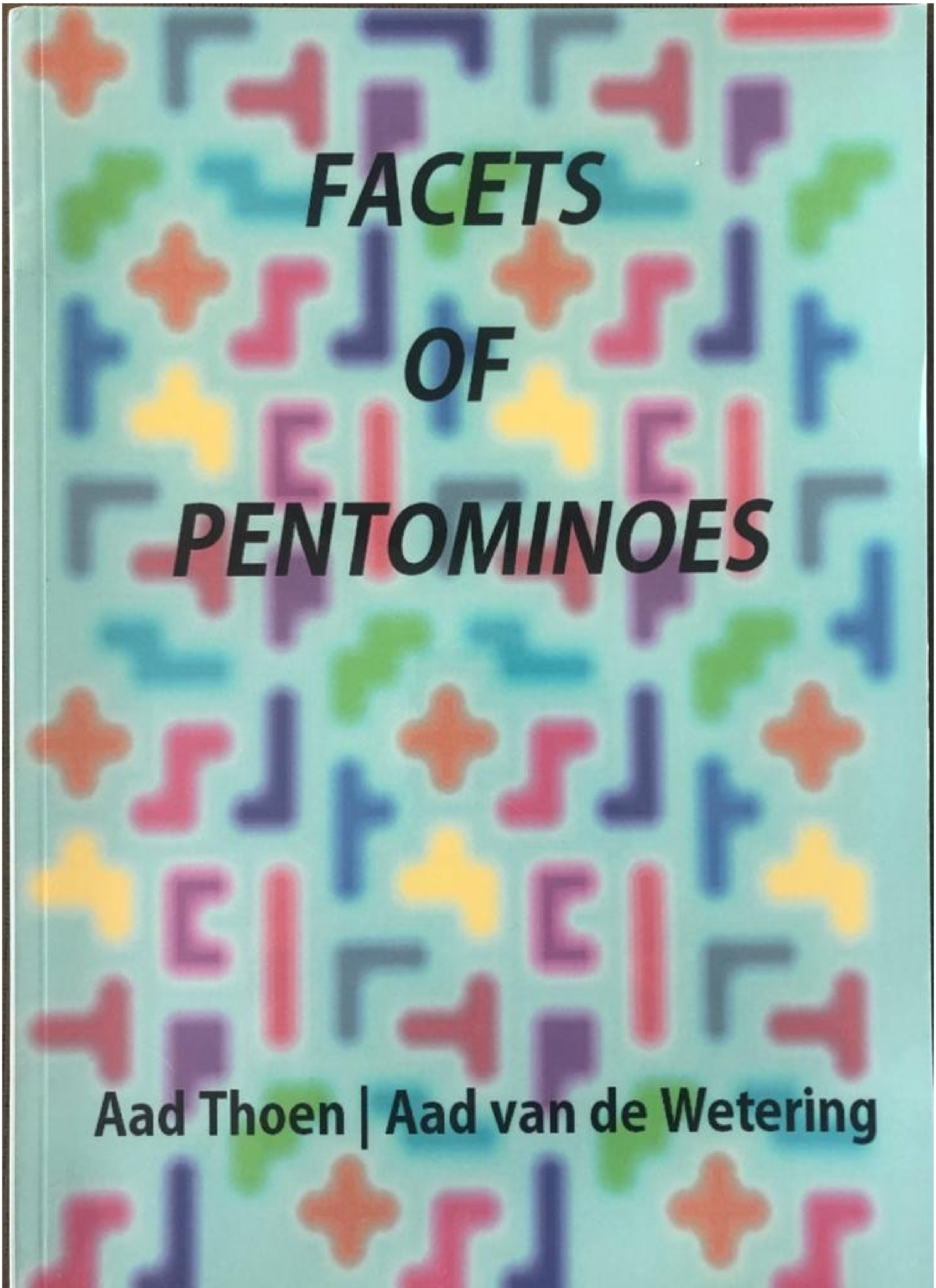
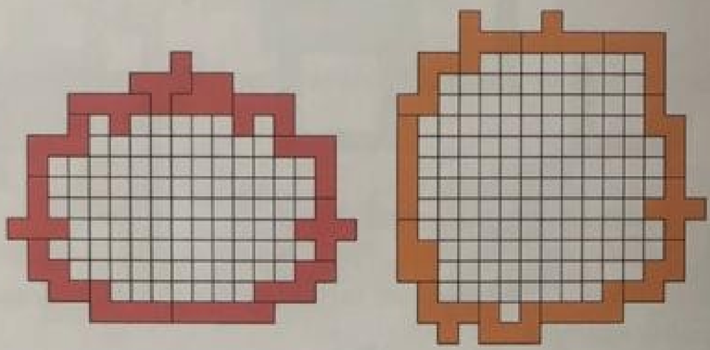

FACETS
OF
PENTOMINOES
Aad Thoen | Aad van de Wetering

5.4. Symmetric enclosures of large areas

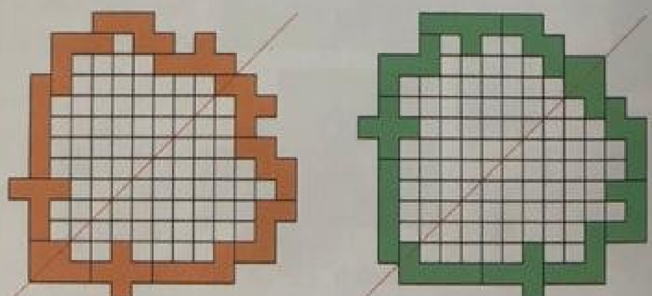
When you see the symmetric enclosure (left below) by Rodolfo Kurchan from Argentina for the first time you can only be amazed. How on earth did he do that using a complete pento set? Not only the inside but the outside as well is symmetric in the vertical axis and he managed to enclose 97 squares!



Symmetric resp. free enclosures by a pento set of 97 resp. 128 squares.

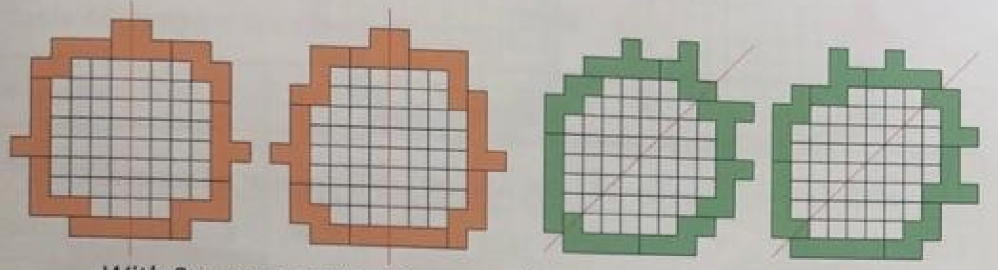
When no restrictions are imposed on the enclosure a maximum of 128 squares can be enclosed. As a matter of fact Odette's very first contest (Wedstrijd 1) was about enclosing as many as possible squares by a pento set.

When one day in 2016 one of us (AT) tried to find diasymmetric enclosures the result left below was reason enough for Odette to start a contest on finding symmetric enclosures, with any ortho- or diagonal axis, by 2–12 pentos. This contest was quite exciting and we had to fight hard against George while Helmut joined later with a catch-up game.

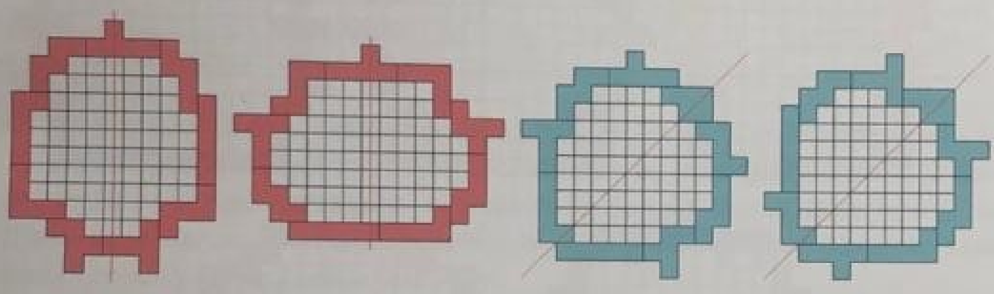


Diasymmetric enclosures by 11 pentos of 85 (AT) *resp. 93 squares* (AW).

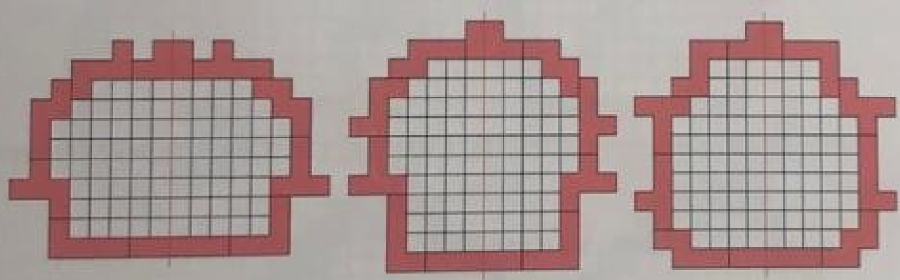
Soon AW could improve AT's 11-pento enclosure from 85 to 93 squares and later Helmut would even do better with 98 squares. The final results of the contest are rendered on Odette's website (Potpourri, problem 68). Here we will concentrate on the results for 8–12 pentos. A thin red line indicates the axis of symmetry (abbreviated: ortho or dia).



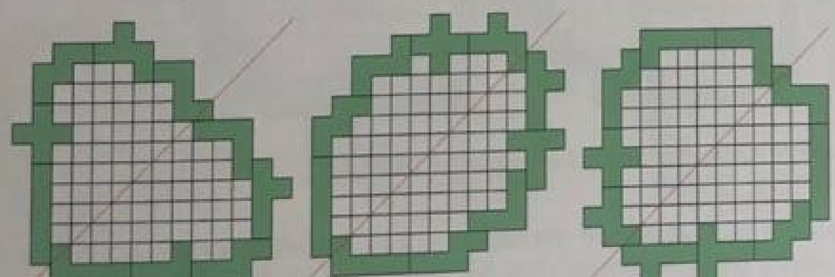
With 8 pentos: ortho: 58 squares (AW, HP), *dia: 54 squares* (OM, HP).



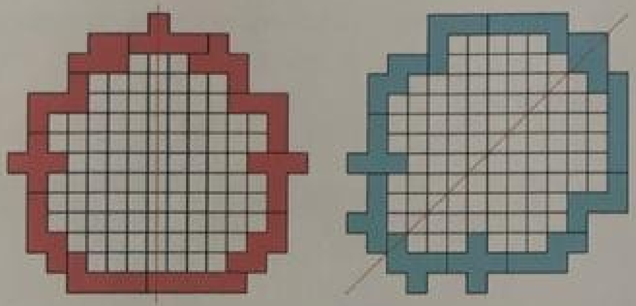
With 9 pentos: ortho: 72 squares (GS, HP), *dia: 66 squares* (GS, HP).



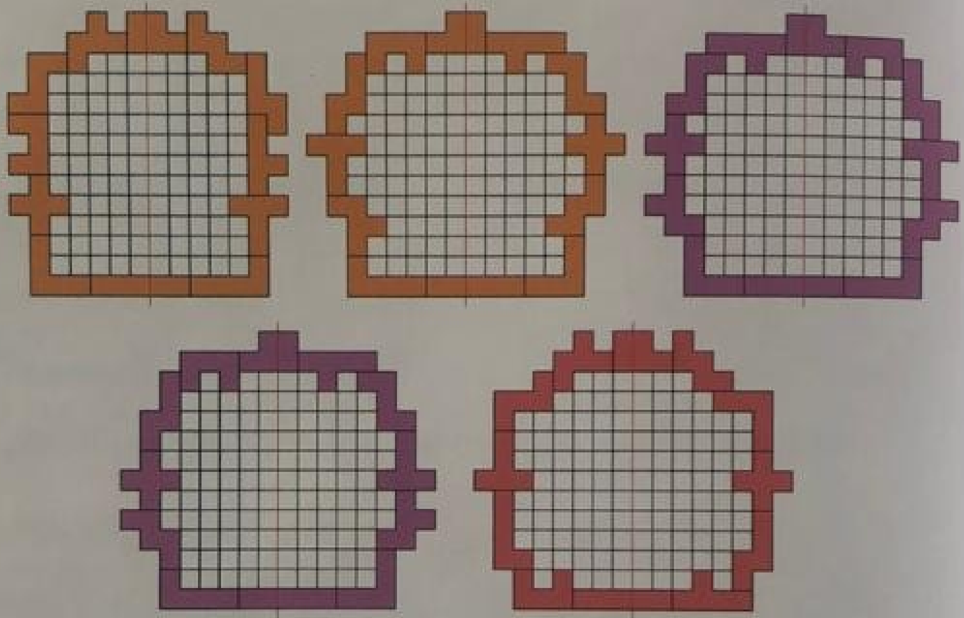
With 10 pentos: ortho: 84 (GS), *86* (AT) *resp. 88* (HP) *squares.*



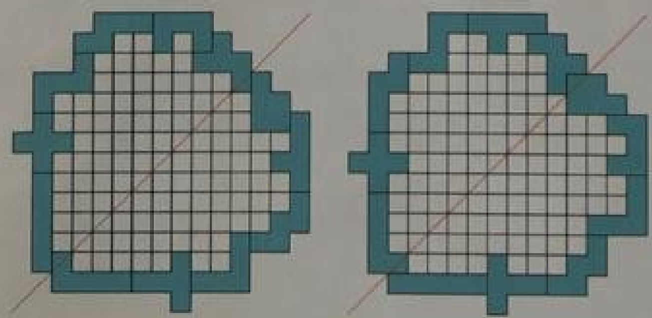
With 10 pentos: dia: 76 (AW), *78* (GS) *resp. 83* (HP) *squares.*



With 11 pentos: ortho: 97 (OM, AW, HP) *squares, dia: 98 squares* (HP).



With 12 pentos: ortho: 104 (AT), *110* (GS), *114* (OM, MR, HP) *squares.*



With 12 pentos: dia: 108 squares (AW, HP).

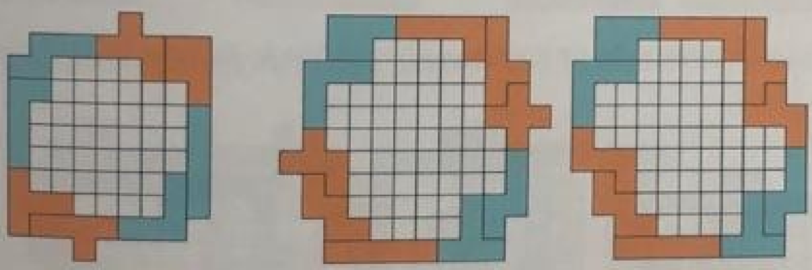
It was quite a feat when we managed in succession to break Rodolfo's record, Odette with a staggering 114 squares. Rodolfo however informed her that this record was already gained by Mike Reid in 1995. Note that Odette's solution resembles MR's a lot, Helmut came up with a rather different one.

The next table sums up the numbers of enclosed squares by 2–12 pentos. For the arbitrary case refer to Wedstrijd 9 and for the point-symmetric case to Potpourri, problem 69 on Odette's website.

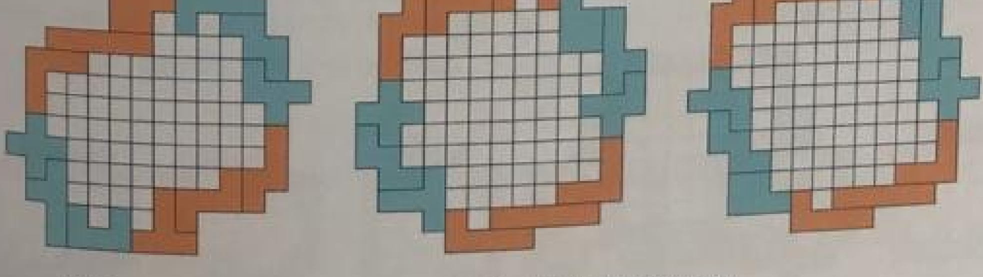
Table of enclosed areas by 2–12 pentos

# pentos	2	3	4	5	6	7	8	9	10	11	12
arbitrary	1	6	13	24	34	47	62	78	94	111	128
orthosymm.	1	6	13	23	32	46	58	72	88	97	114
diasymm.	1	4	11	21	32	41	54	66	83	98	108
point-symm.	-	2	8	10	18	20	41	34	55	50	69

Of the point-symmetric case we mention here the even cases of enclosure by 8, 10 and 12 pentos. The trick to produce them is by placing 22 and/or 222 congruences in opposite positions, necessarily to do so it seems. Note how close these congruences are in the choices by AW and HP. In the 12-pento case posthumously Pieter Torbijn takes part.



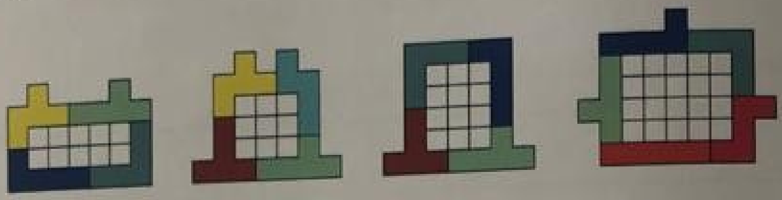
With 8 pentos: 41 (AT), *with 10 pentos: 54* (AW), *55* (HP) *squares.*



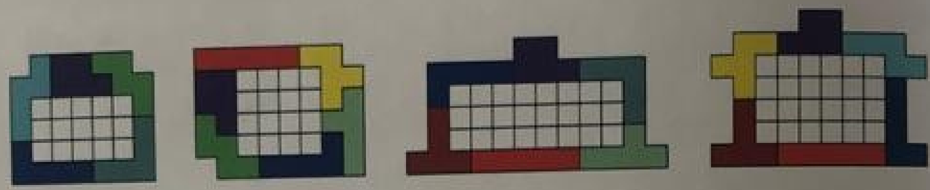
With 12 pentos: 64 (PT), *69* (AW), *74* (HP) *squares.*

We treat the uneven point-symmetric case in the next chapter on multiple enclosures since in that case the enclosed area consists of two parts.

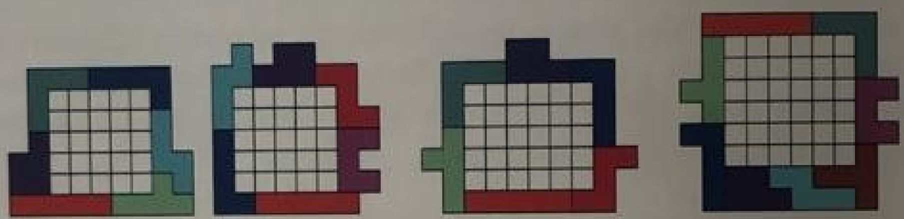
We close this chapter with a few symmetric enclosures of rectangles, there are a lot more. It seems that larger than a 7×7 square is impossible.



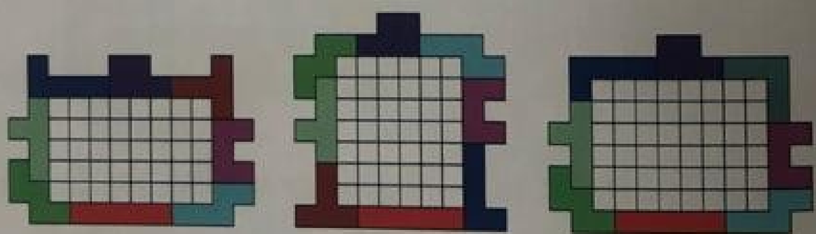
Symmetric enclosures of 2×5, 3×3, 3×4 resp. 4×5 by 4 resp. 5 pentos.



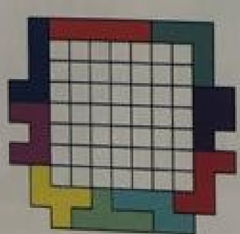
Symmetric enclosures of 3×5, 4×4, 3×8 resp. 4×6 by 5 or 6 pentos.



Symmetric enclosures of 5×5 (twice), 5×6 resp. 6×6 by 6 resp. 8 pentos.



Symmetric enclosures of 5×8, 6×7 resp. 6×8 by 8 pentos.



Symmetric enclosure of 7×7 by 9 pentos.