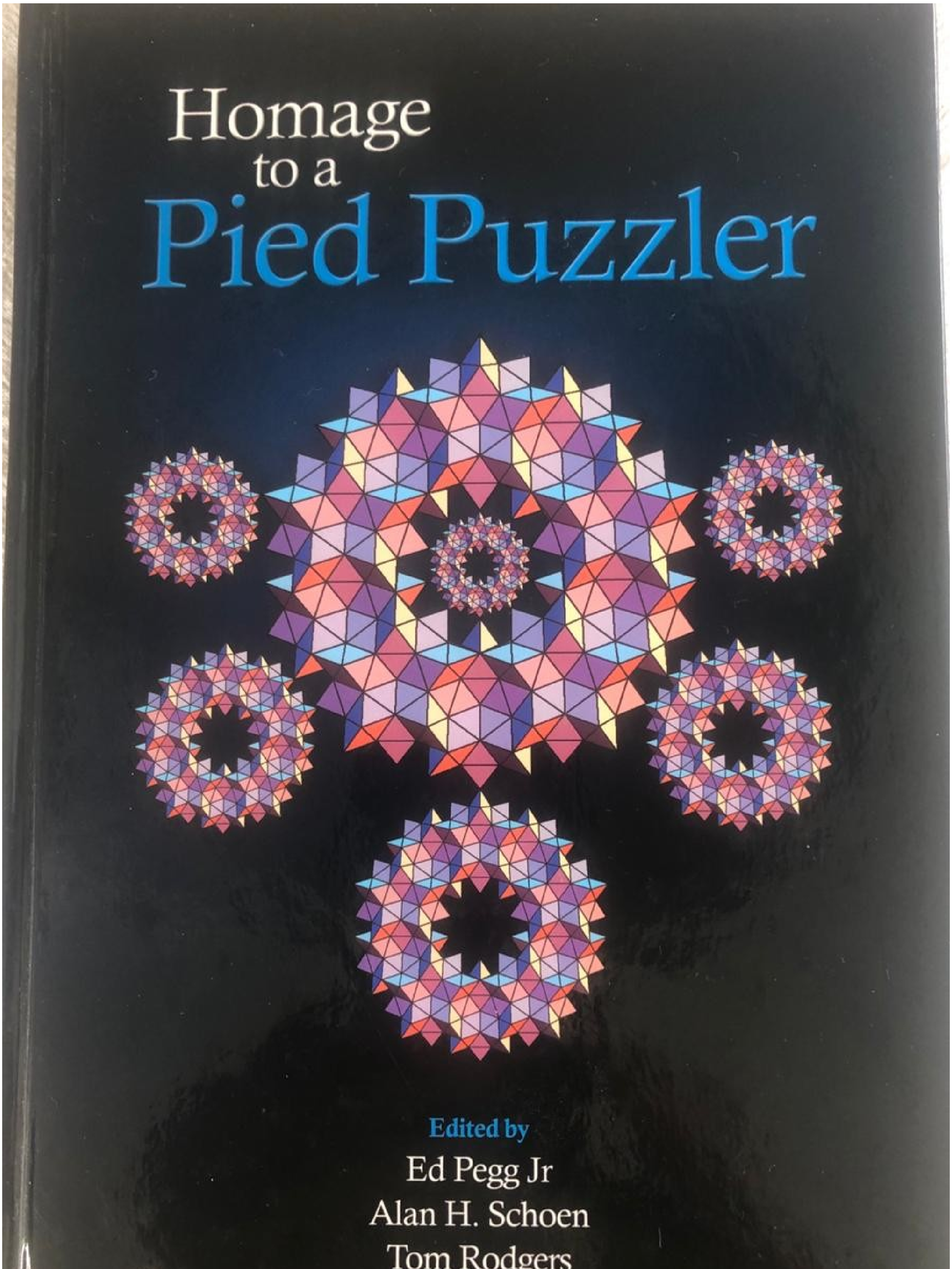

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Variations of the 14-15 Puzzle

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The goal of the 14-15 problem is to go from position A to position B by sliding numbers to empty squares.

1	2	3	4
5	6	7	8
9	10	11	12
13	15	14	

Position A

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

Position B

Everybody knows that there is no solution to this problem.

Here I present seven variations of the 14-15 Puzzle, all of which do have solutions.

1. *The 14-15 multiples exchange.* You go from position A to position B by exchanging numbers whose sum is a multiple of 14 or 15 (*cf.* the name of this puzzle).

   Example:

1	2	3	4
5	**8**	7	**6**
9	10	11	12
13	15	14	

1	2	3	4
5	8	7	**9**
6	10	11	12
13	15	14	

You can exchange numbers 6 and 8 because $6+8=14$. Then you can exchange 6 and 9, for example, because $6+9=15$.

2. *Square exchanges.* You go from position A to position B by exchanging numbers whose sum is a square number.

Example:

3	2	**1**	4
5	6	7	8
9	10	11	12
13	15	14	

13	2	1	4
5	6	7	8
9	10	11	12
3	15	14	

You can exchange numbers 1 and 3 because $1+3=4$; then, for example, you can exchange 3 and 13, because $3+13=16$.

3. *Neighbors sum.* Go from position A to position B by moving numbers to an empty space, subject to the rule that you cannot move a number if the sum of itself and its neighbors in the same row or column (not diagonal) is 40 or more.

Example:

	2	3	4
5	6	7	8
9	10	11	12
13	15	14	**1**

1	2	3	4
5	6	7	8
9	10	11	12
13		14	**15**

You can move the number 1 because the sum $1+12+14=27$ is less than 40, but you cannot move 15 because $15+14+12=41$.

4. *Neighbors multiples.* In this case you can move a number as long as the sum of itself and its neighbors in the same row or column (not diagonal) is:

   (a) a multiple of 4.

   (b) (Is it possible to replace multiples of 4 by multiples of 3?)

Example: This is the solution for multiples of 2:

1	2	3	4
5	6	7	8
9	10	11	12
13	15		**14**

(1) 14 (12 + 14 = 26)

1	2	3	4
5	6	7	8
9	10	11	12
13		**15**	14

(2) 15 (15 + 11 + 14 = 40)

1	2	3	4
5	6	7	8
9	10	11	12
13	**14**	15	

(3) 14(10 + 13 + 14 + 15 = 52)

Total = 3 moves

5. *Strip.* Moving any of the four rows or four columns (they move as a continuous line with no end), go from position A to position B with the smallest possible number of moves.

Example:

	E	**F**	**G**	**H**
A	1	2	3	4
B	5	6	7	8
C	9	10	11	12
D	13	15	14	

	E	**F**	**G**	**H**
A	1	2	3	4
B	**6**	**7**	**8**	**5**
C	9	10	11	12
D	13	15	14	

B 3 = means move.row B 3 times to the right.

6. *Moving numbers.* A counter can move (horizontally or vertically, but not diagonally) a number of squares equal to itself, so you can only move numbers 1, 2, or 3 (with 4 you would leave the board) and exchange positions with another counter.

How many moves do you need to go from position A to position B?

Example:

1	**4**	3	**2**
5	6	7	8
9	10	11	12
13	15	14	

5	4	3	2
1	6	7	8
9	10	11	12
13	15	14	

Example: Number 2 moves to the right and exchanges with 4. Number 1 moves down and exchanges with 5.

7. *Jumping.* Counter movement is implemented by jumping over one or two counters (horizontally, vertically, or diagonally) to an empty space.

   How many moves do you need to go from position A to position B?

   Example:

1	2	3	
5	6	7	8
9	10	11	12
13	15	14	**4**

Number 4 jumps over 8 and 12.

1		3	**2**
5	6	7	8
9	10	11	12
13	15	14	4

Number 2 jumps over 3.

Open Problems

You can extend these problems to 5×5 or other boards.

Solutions

1. *The 14-15 multiples exchange:*

   14/1, 13/1, 13/15, 13/1, 14/1 = 5 moves

2. *Square exchanges:*

   14/11, 11/5, 5/4, 4/12, 12/13, 13/3, 3/1, 15/1, 1/3, 3/13, 13/12, 12/4, 4/5, 5/11, 11/14 = 15 moves

3. *Neighbors sum:*

   13, 14, 15, 14, 13 = 5 moves

4. *Neighbors multiples:*

   (a) 2, 14, 15, 14, 2 = 5 moves

1	2		4
5	6	7	8
9	10	11	12
13	14	15	3

   (b) Near solution: 13, 15, 14, 4, 13, 8, 4, 15, 13, 8, 3, 4, 8, 4 = 14 moves (only 3 is not in position)

5. *Strip:*

   G1, D2, G3, D1, G1, D1, G3, D1 = 8 moves

6. *Moving numbers:*

   3/14, 1/2, 1/14, 3/1, 1/15, 2/3, 3/13, 1/3, 3/14, 2/11, 2/9, 2/13, 1/13, 2/1, 2/9, 2/11, 2/1, 1/3, 1/2 = 19 moves

7. *Jumping:*

   8, 15, 8, 13, 5, 15, 13, 5, 14, 15, 14, 5, 13, 14, 5, 13, 8, 14, 8 = 19 moves